Source : http://www.angelfire.com (site disparu vers 2014). Auteur inconnu.

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Conventions Used
* = multiplication [2 * 3 = 6]
^ = to-the-power-of [2 ^ {3} = 8, curly brackets {} show the power]
PI used is 3.14159265358979 [Circumference of Circle = 2*PI*radius]
Converting between FM synths
Direct conversion between the different FM synths can be a bit tricky. Below are some conversion tables for the various FM synths. Please note that there are limitations to conversion (eg Conversion from 6-op to 4-op ~or~ from a complex Rate/Level envelope to ADSDR may not be ideal).

MODULATION OUTPUT CONVERSION

Orig	:	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75
X	:	3	6	11	15	19	23	28	33	38	43	48	53	58	63	68
CX	:	21	32	42	49	54	59	64	69	74	79	84	89	94	99	104

continued...

Orig	:	80	82	84	86	88	90	91	92	93	94	95	96	97	98	99
X	:	73	75	77	79	81	83	84	85	86	87	88	89	90	91	92
CX	:	109	111	113	115	117	119	120	121	122	123	124	125	126	127	127

ENVELOPE PARAMETERS

Attack (A) ~ Rate (R) Conversion

DX-7 R	:	15	21	27	34	40	47	54	60	67	74	80	85	89	93	96	99
DX-21 A	:	1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31

Decay (D) ~ Rate (R) Conversion

DX-7 R	:	10	16	21	27	33	39	45	51	57	63	69	75	81	87	93	99
DX-21 D	:	1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31

Sustain (S) ~ Level (L) Conversion

DX-7 L	:	35	39	44	48	53	57	62	66	71	75	80	84	89	93	99
DX-21 S	:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Release (R bottom) ~ Rate (R top) Conversion

DX-7 R	:	21	27	32	38	43	49	54	60	65	71	76	82	87	94	99
DX-21 R	:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Certain parameters like Feedback and Frequency are the same (although Frequency may take some fiddling to convert). However, most other parameters are not the same.

Enharmonic Detuned Frequencies

The frequency of an operator is dependent on 3 parameters (1) the Coarse Frequency, (2) the Fine Frequency, and (3) Detune. So far, we have dealt mainly with the Coarse Frequency which is the main integer ratio of the base frequency (eg M:C = 2:1). We have also looked at Detune and its effects where (a) detuning the Carrier shifts the entire spectrum, and (b) detuning the Modulator changes the separation between the sidebands.

FM synths will have some form of Fine Frequency. The Fine Frequency allows the selection of non-integer multiples of the base frequency. Normally, this would yield something clangorous or enharmonic. This brings another dimension into FM sounds because a new set of overtones are introduced. Typically, this would be used for bell-type or percussion sounds. You can also obtain very unique and strange timbres too (a great source of experimentation). Specifically for the original DX-7 (and DX-9) range, the Fine Frequency can also be used to obtain "extra" detuning for the operator. Basically the Fine Frequency had a range of 0~99 where each increment increased the Frequency by 1 percent. The table below shows the frequencies which are within 0.05 of a Coarse frequency; achieved by a combination of the CF [Coarse Frequency selected] and FF [Fine Frequency increments].

Freq	CF	FF	Freq	CF	FF	Freq	CF	FF	Freq	CF	FF	Freq	CF	FF
0.505	0.5	1	4.95	3	65	11.97	7	71	18.96	12	58	25.95	15	73
0.510	0.5	2	4.96	4	24	11.97	9	33	18.98	13	46	25.96	22	18
0.515	0.5	3	4.98	3	66	11.99	11	9	19.03	11	73	25.99	23	13
0.520	0.5	4	5.01	3	67	12.04	7	72	19.04	14	36	26.01	17	53
0.525	0.5	5	5.04	4	26	----	--	--	19.04	16	19	26.03	19	37
0.530	0.5	6	5.04	3	68	12.95	7	85	19.04	17	12	26.04	14	86
0.535	0.5	7	5.05	5	1	12.96	9	44	19.05	15	27	26.04	21	24
0.540	0.5	8	----	--	--	12.96	8	62	----	--	--	----	--	--
0.545	0.5	9	5.95	5	19	12.96	12	8	19.95	19	5	26.98	19	42
0.550	0.5	10	5.96	4	49	12.98	11	18	19.95	15	33	27.02	14	93
----	---	--	5.97	3	99	13.02	7	86	19.98	18	11	27.03	17	59
0.950	0.5	90	6.04	4	51	13.04	8	63	20.02	11	82	27.04	16	69
0.955	0.5	91	6.05	5	21	13.05	9	45	20.02	13	54	27.04	26	4
0.960	0.5	92	----	--	--	----	--	--	20.02	14	43	----	--	--
0.965	0.5	93	6.95	5	39	13.95	9	55	20.04	12	67	28.05	17	65
0.970	0.5	94	6.96	6	16	13.97	11	27	----	--	--	28.05	15	87
0.975	0.5	95	6.96	4	74	14.04	12	17	20.96	16	31	----	--	--
0.980	0.5	96	7.02	6	17	14.04	9	56	21.01	11	91	28.95	15	93
0.985	0.5	97	7.04	4	76	14.04	13	8	----	--	--	28.96	16	81
0.990	0.5	98	7.05	5	41	----	--	--	21.96	12	83	28.98	18	61
0.995	0.5	99	----	--	--	14.95	13	15	21.96	18	22	28.98	21	38
1.01	1	1	7.95	5	59	14.96	11	36	21.97	13	69	28.98	23	26
1.02	1	2	7.96	4	99	14.96	8	87	21.98	14	57	29.04	24	21
1.03	1	3	7.98	6	33	14.98	14	7	22.04	19	16	29.04	22	32
1.04	1	4	7.98	7	14	15.03	9	67	22.05	15	47	----	--	--
1.05	1	5	8.04	6	34	15.04	8	88	22.05	21	5	29.96	28	7
----	---	--	8.05	5	61	----	--	--	----	--	--	29.97	27	11
1.95	1	95	8.05	7	15	15.95	11	45	22.95	15	53	30.02	19	58
1.96	1	96	----	--	--	15.96	12	33	22.95	17	35	30.03	21	43
1.97	1	97	8.95	5	79	15.96	14	14	22.96	14	64	----	--	--
1.98	1	98	8.96	7	28	15.99	13	23	22.99	19	21	30.96	18	72
1.99	1	99	8.96	8	12	16.02	9	78	23.01	13	77	30.96	24	29
2.02	2	1	9.03	7	29	16.05	15	7	23.04	12	92	30.97	19	63
2.04	2	2	9.04	8	13	----	--	--	23.04	16	44	31.02	22	41
----	---	--	9.05	5	81	16.95	15	13	23.04	18	28	31.03	29	7
2.96	2	48	----	--	--	16.96	16	6	----	--	--	31.04	16	94
2.98	2	49	9.95	5	99	17.01	9	89	23.97	17	41	31.05	27	15
3.02	2	51	9.96	6	66	17.03	13	31	23.98	22	9	31.05	23	35
3.03	3	1	9.99	9	11	17.04	12	42	24.05	13	85			
3.04	2	52	10.01	7	43	17.05	11	55	----	--	--			
----	---	--	10.02	6	67	----	--	--	24.96	13	92			
3.96	2	98	----	--	--	18.02	17	6	24.96	16	56			
3.96	3	32	10.96	8	37	18.04	11	64	24.96	24	4			
3.98	2	99	10.98	6	83				24.99	17	47			
3.99	3	33	10.98	9	22				24.99	21	19			
4.02	3	34	10.99	7	57				25.02	18	39			
4.04	4	1	11.04	6	84				25.05	15	67			
4.05	3	35	11.04	8	38									

These "near integer" or "extra detuned" frequencies are not available for the other DX-21 variants nor the CX variants.

About DX Operators

A DX-Synth actually calculates the entire algorithm and operator arrangement and outputs the final waveform calculation into a D/A converter for conversion into voltage. The entire process is handled digitally (ie it's one massive calculation). Each operator has 2 inputs : (1) Pitch Data input, and (2) Modulation Data input. Both information is handled by an input buffer which supplies this information to an Oscillator (Waveform calculator). This Waveform calculation is further processed by an Amplifier (Magnitude calculator) which is controlled from an Envelope Generator. The destination of the final out put depends on the position of the operator in an algorithm. If it is a modulator, then the information is passed down thechain. If it is a carrier, then the information is ready for D/A conversion (actually it's not quite ready as there may be more carriers in the algorithm which then need to be summed together).

Modulation Index Calculation

We already know about how sidebands are generated from a M:C combination. The amplitudes of each order of Sideband is determined by the Modulator's Output
level. Before we can work out the amplitudes, we need to convert the Modulator's Output into a reference calculation known as Modulation Index.
The first step is to convert Output level into a TL number (doesn't apply to CX-5 and FB-01... see below). The reason for this is that the Output is non-linear and
actually goes through a look-up table. So to get a proper linear output, we have to look-up the correct TL number [TL numbers have a negative relationship to the
output levels].
For all Output values beyond 19, use the formula :- [TL] = 99 - [Out]
For Output values from 0 to 19, use this table :

DX.Out	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
TL.vall	127	122	118	114	110	107	104	102	100	98	96	94	92	90	88	86	85	84	82	81

Now we can convert it into Modulation Index. The formula for conversion differs for each ganeration of FM-synth and are as follows:-
For the classic/ original range of DX-synths ~ DX-1/DX-5/DX-7/DX-9/TX-7.
MODULATION INDEX I = (PI) * 2to-the-power-of((33/16) - (TL/8))
For the next generation DX-synths ~ DX-21/DX-27/DX-100.
MODULATION INDEX I =(8 x PI) *2 to-the-power-of(-TL / 8)
For the computer range CX-5/FB-01 (No conversion into TL numbers).
MODULATION INDEX I =(8 x PI) * 2to-the-power-of((OUT-135) / 8)
NOTE - In case you want to convert from "I" back into TL values (or OUT values), the trick is to take LOGS to-the-base 2 on each side of theequation.
NOTE - Don't forget that the output is controlled from an Envelope Generator. This means that changes in Level settings over time (ie an envelope shape) results in a change in "I" (Mod Index). The original DX-synths have envelope Levels from 0~99.

DX Output ~to~ Modulation Index Table

Below is the table of Modulation Indices for each of the FM synth types:-

		------Modulation Indices--------------------					------Modulation Indices-------------------------------			Mod Indx-	
OUT	TL	--[DX7]--	--[DX21]-	--[CX5]--	OUT	TL	--[DX7]--	--[DX21]-	--[CX5]--	OUT	--[CX5]--
0	127	0.000218	0.000418	0.000209	50	49	0.188025	0.360107	0.015915	100	1.211250
1	122	0.000337	0.000645	0.000228	51	48	0.205043	0.392699	0.017355	101	1.320877
2	118	0.000476	0.000912	0.000249	52	47	0.223601	0.428241	0.018926	102	1.440427
3	114	0.000674	0.001290	0.000271	53	46	0.243838	0.467001	0.020639	103	1.570796
4	110	0.000952	0.001824	0.000296	54	45	0.265907	0.509268	0.022507	104	1.712966
5	107	0.001235	0.002366	0.000322	55	44	0.289974	0.555360	0.024544	105	1.868002
6	104	0.001602	0.003068	0.000352	56	43	0.316219	0.605625	0.026765	106	2.037071
7	102	0.001905	0.003648	0.000383	57	42	0.344839	0.660439	0.029188	107	2.221441
8	100	0.002265	0.004339	0.000418	58	41	0.376050	0.720213	0.031829	108	2.422499
9	98	0.002694	0.005160	0.000456	59	40	0.410085	0.785398	0.034710	109	2.641754
10	96	0.003204	0.006136	0.000497	60	39	0.447201	0.856483	0.037852	110	2.880853
11	94	0.003810	0.007297	0.000542	61	38	0.487676	0.934001	0.041277	111	3.141593
12	92	0.004531	0.008678	0.000591	62	37	0.531815	1.018535	0.045013	112	3.425931
13	90	0.005388	0.010319	0.000645	63	36	0.579948	1.110721	0.049087	113	3.736004
14	88	0.006408	0.012272	0.000703	64	35	0.632438	1.211250	0.053530	114	4.074142
15	86	0.007620	0.014594	0.000767	65	34	0.689679	1.320877	0.058375	115	4.442883
16	85	0.008310	0.015915	0.000836	66	33	0.752100	1.440427	0.063658	116	4.844998
17	84	0.009062	0.017355	0.000912	67	32	0.820171	1.570796	0.069420	117	5.283508
18	82	0.010776	0.020639	0.000995	68	31	0.894403	1.712966	0.075703	118	5.761706
19	81	0.011752	0.022507	0.001085	69	30	0.975353	1.868002	0.082555	119	6.283185
20	79	0.013975	0.026765	0.001183	70	29	1.063630	2.037071	0.090027	120	6.851862
21	78	0.015240	0.029188	0.001290	71	28	1.159897	2.221441	0.098175	121	7.472009
22	77	0.016619	0.031829	0.001407	72	27	1.264876	2.422499	0.107060	122	8.148283
23	76	0.018123	0.034710	0.001534	73	26	1.379357	2.641754	0.116750	123	8.885766
24	75	0.019764	0.037852	0.001673	74	25	1.504200	2.880853	0.127317	124	9.689996
25	74	0.021552	0.041277	0.001824	75	24	1.640341	3.141593	0.138840	125	10.567016
26	73	0.023503	0.045013	0.001989	76	23	1.788805	3.425931	0.151406	126	11.523413
27	72	0.025630	0.049087	0.002169	77	22	1.950706	3.736004	0.165110	127	12.566371
28	71	0.027950	0.053530	0.002366	78	21	2.127260	4.074142	0.180053		
29	70	0.030480	0.058375	0.002580	79	20	2.319793	4.442883	0.196350		
30	69	0.033238	0.063658	0.002813	80	19	2.529752	4.844998	0.214121		
31	68	0.036247	0.069420	0.003068	81	18	2.758714	5.283508	0.233500		
32	67	0.039527	0.075703	0.003346	82	17	3.008399	5.761706	0.254634		
33	66	0.043105	0.082555	0.003648	83	16	3.280683	6.283185	0.277680		
34	65	0.047006	0.090027	0.003979	84	15	3.577610	6.851862	0.302812		
35	64	0.051261	0.098175	0.004339	85	14	3.901411	7.472009	0.330219		
36	63	0.055900	0.107060	0.004731	86	13	4.254519	8.148283	0.360107		
37	62	0.060960	0.116750	0.005160	87	12	4.639586	8.885766	0.392699		
38	61	0.066477	0.127317	0.005627	88	11	5.059505	9.689996	0.428241		
39	60	0.072494	0.138840	0.006136	89	10	5.517429	10.567016	0.467001		
40	59	0.079055	0.151406	0.006691	90	9	6.016799	11.523413	0.509268		
41	58	0.086210	0.165110	0.007297	91	8	6.561366	12.566371	0.555360		
42	57	0.094012	0.180053	0.007957	92	7	7.155220	13.703724	0.605625		
43	56	0.102521	0.196350	0.008678	93	6	7.802823	14.944017	0.660439		
44	55	0.111800	0.214121	0.009463	94	5	8.509039	16.296566	0.720213		
45	54	0.121919	0.233500	0.010319	95	4	9.279172	17.771532	0.785398		
46	53	0.132954	0.254634	0.011253	96	3	10.119009	19.379993	0.856483		
47	52	0.144987	0.277680	0.012272	97	2	11.034858	21.134032	0.934001		
48	51	0.158110	0.302812	0.013383	98	1	12.033598	23.046825	1.018535		
49	50	0.172420	0.330219	0.014594	99	0	13.122731	25.132741	1.110721		

Note - When Output=0, it should really be Index=Zero. But these are the equations given and so we'll use these figures anyway.

Deviation

Before we calculate the amplitudes of the sidebands, you might want to know what the Modulation Index actually is.

FM is a Modulator modulating the Pitch of a Carrier. Think of the Carrier being a "centre frequency" and think of the Modulation as "shifting the carrier to up and down in terms of pitch" (frequency). This occurs over time at the rate "M" (the Modulator frequency). This shifting of the carrier up and down is sinusoidal over time (since the Modulator is a Sine wave).

The Deviation is the difference between Carrier and the the lowest or highest instantaneous frequency. Let's say the Carrier is at 100Hz. The shifting could cause the Carrier to oscillate between say 90Hz (lowest freq) and 110Hz (highest freq). The width of oscillation from the Carrier to either extreme is the Deviation, in this case, 10Hz.

The relationship to Modulation Index is as follows:-

"d" = Deviation

"I" = Modulation Index $\quad I = \frac{d}{M} \quad$ ~or~ $\quad d = I * M$

"M" = Modulator Frequency

So we can think of the Modulation Index as the "change in pitch" relative to the Modulator pitch (frequency).

Bessel Functions

"Order" refers to the distance of any Sideband from the Carrier expressed in multiples of "M". So C+M and C-M are the first order sidebands, C+2M and C-2M are the second order sidebands... etc.

Given you know "I" (Mod.Index), you can calculate the amplitudes of each Sideband generated by "C" and "M" using a Bessel Function. We use the letter "J" to denote a Bessel function, and they are ordered as J0, J1, J2, J3, J4 etc

Bessel function J0(I) = Amplitude for the Carrier, "C".

Bessel function Jn(I) = Amplitude for the Sidebands at "C + nM" and "C - nM" (where n = 1, 2, 3, 4... etc).

Jn(I) (the nth order of "J") is a Bessel function of "I" (index).

The Bessel function is expressed as:-

$$Jn(index) = \sum_{k=0}^{inf} \frac{(-1)^{k} * (index/2)^{n+2k}}{k! * (n + k)!}$$

Also: Jn+1(index) = (2n/index) * Jn(index) -Jn-1(index)

I shall not even pretend to understand the logic for Bessel functions. However, note that the Bessel function is in 2 parts: The sum (sigma) portion and the algebra portion. This means that you start with k=0 in the algebra portion, then do it again for k=1... then k=2, up to k=infinity. Jn(index) is the sum of all these numbers.

Basically, each increase in "k" takes the sum one step closer to the final answer. Thankfully, there comes a point where further increase in "k" becomes fairly insignificant. Let's look at an example:-

Calculating the zero order (n=0), where Modulation Index = 1

k=0, $$\frac{(-1)^{0} * (1/2)^{0+2*0}}{0! * (0+0)!} = \frac{1}{1} = 1$$

k=1, $$\frac{(-1)^{1} * (1/2)^{0+2*1}}{1! * (0+1)!} = \frac{-0.25}{1} = -0.25$$

k=2, $$\frac{(-1)^{2} * (1/2)^{0+2*2}}{2! * (0+2)!} = \frac{0.0625}{4} = 0.015625$$

Sum for k=0 to 2, we obtain1 - 0.25 + 0.015625 = 0.765625

If we were carry on with more "k", we should get0.765197684eventually.

Fortunately, some spreadsheets include this feature [in Excel, the formula is "=BesselJ(index,order)" - you will need to activate the Analysis Toolpack under
Tools/AddIns]. The Bessel function tables can be found near the end of this article.

Spectrum Amplitudes

Having calculated the Bessel functions, you now have to assign the Jn(index) to the relevant harmonics as follows:-

Freq :	C	C+M	C+2M	C+3M	C+4M	C+5M	C+6M	etc
Amplitude :	J0(I)	J1(I)	J2(I)	J3(I)	J4(I)	J5(I)	J6(I)	etc
Freq :		C-M	C-2M	C-3M	C-4M	C-5M	C-6M	etc
Amplitude:		J1(I)	J2(I)	J3(I)	J4(I)	J5(I)	J6(I)	etc

For a graphical representation of Bessel functions, see FM Spectrum Graphs (contains animated GIFs). It displays the Amplitudes of each Order as the Modulation
Index is increased.

IMPORTANT - Where the Sidebands are reflected (ie "C - nM" becomes negative), the phase is inverted. Instead of treating it as a negative frequency with
amplitude Jn(index), you should treat it as a positive frequency with a negative amplitude Jn(index). In short, reflected Sidebands = inverted phase = negative
amplitude.

This is especially important if you are dealing with conicidental refected Sidebands. If you refer to the previous article "FM Synthesis", the M:C Series of 1:1 and
2:1 (and their permutations) have reflected Sidebands which are coincidental with the non-reflected Sidebands. Where the Sidebands are coincidental, just
remember that the reflected Sidebands are phase inverted and hence will have negative values.

Here's an example of M:C = 2:3 on a DX-7 with Out=75:-

Freq :	3	5	7	9	11	13	etc
Amplitude :	43%	57%	27%	8%	2%	0%	ignore below 0.5%
Freq :		1	1	3	5	7	etc
Amplitude :		57%	-27%	-8%	-2%	-0%	ignore below 0.5%

The resultant spectrum would be:-

Freq :	1	2	3	4	5	6	7	8	9	10	11	etc
Amplitude :	30%	-	35%	-	55%	-	27%	-	8%	-	2%	etc

Note - Bessel functions can in themselves yield negative values (phase inverted). For a DX-7, when Out > 79, some orders are negative. In short, you need to be a
bit careful in assigning amplitude values.

Note - It is important to establish for yourself the number of decimal places to use. This is to define what value of amplitude of Sidebands are significant. I would recommend using either 2 or 3 decimal places (beyond which the Sidebands can be disregarded). The following table gives values below which there are no significant Sidebands (ie they result in Carrier only) :-

No significant Sidebands	index	DX7 Out	DX21 Out	CX5 Out
Using 2 decimal places	0.009	17	12	44
Using 3 decimal places	0.0009	4	2	17

Most of the examples in this article use 2 decimal places as significant (ie anything below 0.005 or 0.5% is ignored).
For a graphical representation of FM Spectrums, see FM Spectrum Graphs (contains animated GIFs). It displays the amplitudes for various M:C combinations as
the Modulation Index is increased. It also shows the effects of reflected Sidebands for coincident and non-coincident M:C combinations.

Two or more Modulators

So far, we have only dealt with M:C ; single sine-modulator to single sine-carrier. When there are two Modulators, they can either be "Two-Into-One" (M1 + M2 :
C) or "In-Series" (M2 : M1: C).

M1 + M2 : C (ie two separate Modulators)

```
M1--->-+->-C
  M2--->-+
```

Basically, you will end up with "M1:C" and "M2:C" added together.
Let's try a DX-7 example with M1+M2:C with 3 + 1 : 1 and let's use M1 Out=70 and M2 Out=80

For M1:C = 3 : 1 with M1 Out=70, we get:-

Freq	:	1	4	7	10	13	16	etc
Amplitude	:	74%	46%	13%	2%	0%	0%	less than 0.5%
Freq	:		2	5	8	11	14	etc
Amplitude:		-46%	-13%	-2%	-0%	-0%less than 0.5%		

For M2:C = 1 : 1 with M2 Out=80, we get:-

Freq	:	1	2	3	4	5	6	7	etc
Amplitude	:	-6%	49%	45%	22%	7%	2%	0%	ignore
Freq	:		0	1	2	3	4	5	etc
Amplitude:			49%	-45%	-22%	-7%	-2%	-0%	ignore

The resultant spectrum would be adding the results:-

Freq	:	1	2	3	4	5	6	7	8	9	10	11	12	13	etc
Amp. M1:C	:	74%	-46%	-	46%	-13%	-	13%	-2%	-	2%	-0%	-	0%	etc
Amp. M2:C	:	-51%	27%	38%	20%	7%	2%	0%	-	-	-	-	-	-	etc
Amplitude	:	23%	-19%	38%	66%	-6%	2%	13%	-2%	-	2%	0%	-	0%	etc

In reality, the overall amplitudes would be factored-down as part of the algorithm calculation so as not to overload the Carrier's input.
Note that it is "convenient" to add up the amplitudes for high amounts of modulation (which results in significant Sidebands) but this is not always correct for the Carrier amplitude. The correct way to approach this is really to think of how much energy is taken from the Carrier and transferred to the Sidebands. For example in M2+M1:C, using M2 Out=17 would result in a Carrier amplitude of 100% and 1st-order Sidebands with amplitudes of less than 1%. In this case, adding up "M2:C" (Carrier amplitude only) with "M1:C" (Carrier and Sidebands) is not representative of what is happening. Don't forget that the Carrier exists unchanged (ie 100%) when there is no modulator.

M2 : M1 : C (ie two Modulators in series)

```
M2--->---M1--->---C
```

This one is a lot more complicated. Basically, "M2:M1" will produce one complex waveform and each sine-frequency (in the harmonic spectrum) will act as a sine-modulator into "C". Let's try a simplified DX-7 example with M2:M2:C with 3 : 2 : 3 and let's use M2 Out=75 and M1 Out=90.
For M2:M1 = 3 : 2 with M2 Out=75, we get J0=43%, J1=57%, J2=27%, J3=8%, and J4=2%.
We can predict the outcome as being:-

Freq	:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	etc
Ampl	:	-57%	43%	-	-27%	57%	-	-8%	27%	-	-2%	8%	-	-0%	2%	etc

So each of these sine-frequencies acts a modulators into "C".
WARNING! The next few calculations make certain assumptions about the inner-workings of the DX-chipsets. Quite frankly, I am not exactly sure of this part.
Let's normalise these sine-frequencies as a factor of M1's output (M1 Out=90).

Freq	:	F1	F2	F4	F5	F7	F8	F10	F11	F13	F14
Ampl	:	-57%	43%	-27%	57%	-8%	27%	-2%	8%	-0%	2%
Ampl*90	:	-51.3	38.7	-24.3	51.3	-7.2	24.3	-1.8	7.2	-	1.8

At this point I would disregard any "Ampl*M1" below 35 as I have precalculated that they would be too small to produce any significant Sidebands (ie Carrier only) for 2 decimal places. Next, we have to convert them into TL numbers linearly. Plus, I'm going to round them to the nearest integer.
TL(F1) would be 127 - (-51.3*127/99) = -61
TL(F2) would be 127 - (38.7*127/99) = 77
TL(F5) would be 127 - (51.3*127/99) = 61
If you compare these TL numbers with the Mod.Index table, we can look up the equivalent Output value (ie Out equivalents are -38, 22 and 38 respectively).

For F1:C = 1:3 with TL(F1)= -61 (similar to DX-Out=-38)

Freq	:	1	2	3	4	5	6	etc
Amplitude	:	-0%	-3%	-100%	-3%	-0%	-0%	ignore

For F2:C = 1:3 with TL(F2)= 77 (similar to DX-Out=22)

Freq	:	1	2	3	4	5	6	etc
Amplitude	:	1%	-	100%	-	1%	-	ignore

For F5:C = 5:1 with TL(F5)= 61 (similar to DX-Out=38):-

Freq	:	1	2	3	4	5	6	7	8	etc
Amplitude	:	-	-1%	100%	-	-	-	-0%	1%	ignore

This results in

Freq	:	1	2	3	4	5	6	7	8	etc
Amplitude	:	1%	-4%	100%	-3%	1%	-0%	-0%	1%	ignore

In this case, there really that much to add up especially since the Carriers are hardly affected (apart from one inverted Carrier in F1:C). As previously mentioned, we have to look at the process as the modulation taking energy away from the Carrier to produce Sidebands. Adding up convenient for Sidebands but not always applicable for the Carrier amplitude.
Using a different M2:M1 would yield a totally result. Also using different modulation outputs would change the Sideband amplitudes greatly. I purposely chose a small output for M2:M1 otherwise the number of significant Sinewave components would be very great indeed. As you can appreciate, the "In-Series" modulators calculation can become very complicated and perhaps confusing too. Furthermore, the results may not quite be what you imagined.

More Modulators and FM Algorithms

The algorithms available on FM synths can be fairly elaborate but, in general, you can analyse them into combinations of "two-into-one" or "in-series". This makes the calculation of frequencies and their amplitudes far too difficult. Perhaps this is why this kind of information is usually not presented at all. You need to ask yourself if this is really worth all the effort.
However, I would recommend scanning through the tables below to get a feel for the numbers of orders generated by the different outputs.
Actual DX algorithms can be found in article Synthesizer Layouts.

FM Bessel Tables

Below are Bessel tables for the DX-7 type, the DX-21 type and the CX-5 type synths. The DX-7 table is calculated to 6 decimal places to illustrate how the amplitudes quickly diminish into insignificance as we progress up the orders. As such, for the DX-21 type and CX-5 type synths, the tables are calculated to 4 decimal places.
Every table contains some small rounding errors. However, the obvious errors will be near Out=Zero where J0=1 and all other orders should be Zero. This arises from the Mod.Index imprecisions.

DX-7 Output level ~to~ Bessel function [Jn] Table - up to 6 decimal places.

OUT	--J0--	--J1--	OUT	--J0--	--J1--	--J2--	OUT	--J0--	--J1--	--J2--
0	1.000000	0.000109	8	0.999999	0.001133	0.000001	18	0.999971	0.005388	0.000015
1	1.000000	0.000168	9	0.999998	0.001347	0.000001	19	0.999965	0.005876	0.000017
2	1.000000	0.000238	10	0.999997	0.001602	0.000001	20	0.999951	0.006987	0.000024
3	1.000000	0.000337	11	0.999996	0.001905	0.000002	21	0.999942	0.007620	0.000029
4	1.000000	0.000476	12	0.999995	0.002265	0.000003	22	0.999931	0.008309	0.000035
5	1.000000	0.000618	13	0.999993	0.002694	0.000004	23	0.999918	0.009061	0.000041
6	0.999999	0.000801	14	0.999990	0.003204	0.000005	24	0.999902	0.009881	0.000049
7	0.999999	0.000952	15	0.999985	0.003810	0.000007	25	0.999884	0.010776	0.000058
			16	0.999983	0.004155	0.000009	26	0.999862	0.011751	0.000069
			17	0.999979	0.004531	0.000010	27	0.999836	0.012814	0.000082
							28	0.999805	0.013974	0.000098

OUT	--JO--	--JI--	--J2--	--J3--	OUT	--J0--	--J1--	--J2--	--J3--	--J4--
29	0.999768	0.015238	0.000116	0.000001	42	0.997792	0.046954	0.001104	0.000017	
30	0.999724	0.016617	0.000138	0.000001	43	0.997374	0.051193	0.001313	0.000022	
31	0.999672	0.018120	0.000164	0.000001	44	0.996878	0.055813	0.001561	0.000029	--J4--
32	0.999609	0.019760	0.000195	0.000001	45	0.996287	0.060846	0.001856	0.000038	0.000001
33	0.999536	0.021547	0.000232	0.000002	46	0.995586	0.066330	0.002206	0.000049	0.000001
34	0.999448	0.023497	0.000276	0.000002	47	0.994752	0.072303	0.002623	0.000063	0.000001
35	0.999343	0.025622	0.000328	0.000003	48	0.993760	0.078808	0.003118	0.000082	0.000002
36	0.999219	0.027939	0.000391	0.000004	49	0.992582	0.085890	0.003707	0.000107	0.000002
37	0.999071	0.030466	0.000464	0.000005	50	0.991181	0.093598	0.004406	0.000138	0.000003
38	0.998896	0.033220	0.000552	0.000006	51	0.989517	0.101984	0.005237	0.000179	0.000005
39	0.998687	0.036223	0.000657	0.000008	52	0.987540	0.111103	0.006224	0.000232	0.000006
40	0.998438	0.039497	0.000781	0.000010	53	0.985191	0.121015	0.007395	0.000301	0.000009
41	0.998143	0.043065	0.000928	0.000013	54	0.982401	0.131782	0.008786	0.000390	0.000013

OUT	--JO--	--JI--	--J2--	--J3--	--J4--	--J5--	--J6--	--J7--	--J8--
55	0.979089	0.143468	0.010437	0.000505	0.000018	0.000001			
56	0.975157	0.156141	0.012395	0.000655	0.000026	0.000001			
57	0.970492	0.169869	0.014718	0.000848	0.000037	0.000001			
58	0.964958	0.184721	0.017469	0.001098	0.000052	0.000002			
59	0.958397	0.200763	0.020728	0.001422	0.000073	0.000003			
60	0.950624	0.218057	0.024585	0.001840	0.000103	0.000005			
61	0.941421	0.236661	0.029144	0.002381	0.000146	0.000007			
62	0.930533	0.256617	0.034527	0.003079	0.000205	0.000011	--J6--		
63	0.917666	0.277953	0.040876	0.003979	0.000290	0.000017	0.000001		
64	0.902478	0.300670	0.048351	0.005140	0.000408	0.000026	0.000001		
65	0.884575	0.324739	0.057135	0.006634	0.000575	0.000040	0.000002		
66	0.863508	0.350080	0.067432	0.008554	0.000810	0.000061	0.000004		
67	0.838770	0.376556	0.079469	0.011019	0.001139	0.000094	0.000006	--J7--	
68	0.809791	0.403950	0.093493	0.014175	0.001601	0.000144	0.000011	0.000001	
69	0.775944	0.431938	0.109763	0.018208	0.002247	0.000221	0.000018	0.000001	
70	0.736553	0.460072	0.128544	0.023345	0.003149	0.000338	0.000030	0.000002	
71	0.690906	0.487735	0.150091	0.029867	0.004405	0.000517	0.000050	0.000004	--J8--
72	0.638284	0.514114	0.174623	0.038109	0.006150	0.000789	0.000084	0.000008	0.000001
73	0.578004	0.538153	0.202292	0.048475	0.008565	0.001201	0.000140	0.000014	0.000001
74	0.509483	0.558520	0.233131	0.061428	0.011893	0.001824	0.000232	0.000025	0.000002
75	0.432336	0.573565	0.266988	0.077491	0.016455	0.002762	0.000384	0.000046	0.000005
76	0.346495	0.581302	0.303438	0.097224	0.022672	0.004169	0.000634	0.000082	0.000009
77	0.252390	0.579415	0.341666	0.121186	0.031077	0.006266	0.001043	0.000148	0.000018
78	0.151156	0.565312	0.380338	0.149858	0.042340	0.009370	0.001708	0.000265	0.000036
79	0.044890	0.536256	0.417441	0.183534	0.057259	0.013929	0.002784	0.000473	0.000070
80	-0.063063	0.489599	0.450135	0.222147	0.076747	0.020556	0.004511	0.000840	0.000136
81	-0.167841	0.423182	0.474638	0.265020	0.101760	0.030074	0.007254	0.001481	0.000262
82	-0.262887	0.335919	0.486208	0.310548	0.133154	0.043537	0.011565	0.002593	0.000504
83	-0.339957	0.228591	0.479313	0.355815	0.171433	0.062226	0.018242	0.004500	0.000960
84	-0.389527	0.104837	0.448134	0.396206	0.216341	0.087561	0.028407	0.007722	0.001810
85	-0.401787	-0.027801	0.387535	0.425130	0.266273	0.120875	0.043550	0.013077	0.003375
86	-0.368493	-0.157054	0.294664	0.434090	0.317518	0.162956	0.065501	0.021791	0.006206
87	-0.285797	-0.265834	0.171203	0.413436	0.363460	0.213275	0.096226	0.035608	0.011220
88	-0.157918	-0.333651	0.026027	0.354228	0.394047	0.268832	0.137294	0.056799	0.019872
89	-0.000902	-0.340427	-0.122499	0.251618	0.396125	0.322743	0.188827	0.087942	0.034318
90	0.155265	-0.273345	-0.246126	0.109719	0.355539	0.363009	0.247787	0.131181	0.057448
91	0.268997	-0.136238	-0.310524	-0.053066	0.261998	0.372510	0.305734	0.186643	0.092507
92	0.297214	0.041363	-0.285652	-0.201051	0.117061	0.331933	0.346843	0.249756	0.141833
93	0.214839	0.201891	-0.163090	-0.285497	-0.056443	0.227627	0.348168	0.307821	0.204132
94	0.039470	0.273200	0.024744	-0.261568	-0.209184	0.064898	0.285453	0.337667	0.270113
95	-0.153442	0.204107	0.197435	-0.118998	-0.274380	-0.117558	0.147691	0.308554	0.317842
96	-0.249334	0.013629	0.252028	0.085997	-0.201036	-0.244935	-0.041017	0.196292	0.312595
97	-0.164935	-0.182076	0.131935	0.229901	-0.006930	-0.234925	-0.205964	0.010948	0.219853
98	0.055158	-0.221098	-0.091905	0.190548	0.186913	-0.066288	-0.241998	-0.175035	0.038361
99	0.213943	-0.043921	-0.220636	-0.023333	0.209968	0.151335	-0.094645	-0.237883	-0.159141

continuing across...

OUT	--J9--	--J10-	--J11-	--J12-	--J13-	--J14-	--J15-	--J16-	--J17-
76	0.000001								
77	0.000002								
78	0.000004	--J10-							
79	0.000009	0.000001							
80	0.000019	0.000002	--J11-						
81	0.000041	0.000006	0.000001						
82	0.000086	0.000013	0.000002	--J12-					
83	0.000180	0.000030	0.000005	0.000001					
84	0.000373	0.000069	0.000011	0.000002	--J13-				
85	0.000765	0.000155	0.000028	0.000005	0.000001				
86	0.001548	0.000344	0.000069	0.000013	0.000002	--J14-			
87	0.003087	0.000755	0.000166	0.000033	0.000006	0.000001	--J15-		
88	0.006043	0.001629	0.000395	0.000087	0.000018	0.000003	0.000001		
89	0.011576	0.003447	0.000921	0.000223	0.000049	0.000010	0.000002	--J16-	
90	0.021585	0.007126	0.002102	0.000561	0.000137	0.000031	0.000006	0.000001	--J17-
91	0.038937	0.014309	0.004679	0.001381	0.000372	0.000092	0.000021	0.000005	0.000001
92	0.067400	0.027723	0.010090	0.003301	0.000982	0.000268	0.000068	0.000016	0.000004
93	0.110760	0.051376	0.020924	0.007621	0.002515	0.000760	0.000212	0.000055	0.000013
94	0.170241	0.090014	0.041333	0.016850	0.006194	0.002077	0.000641	0.000184	0.000049
95	0.239498	0.146743	0.076787	0.035311	0.014543	0.005437	0.001865	0.000591	0.000175
96	0.297977	0.217456	0.131820	0.069136	0.032157	0.013487	0.005164	0.001821	0.000596
97	0.307828	0.282275	0.203778	0.123993	0.065898	0.031273	0.013455	0.005308	0.001936
98	0.226040	0.299753	0.272153	0.197801	0.122344	0.066539	0.032479	0.014432	0.005900
99	0.043850	0.219288	0.290361	0.267496	0.198860	0.126504	0.071062	0.035952	0.016607

continuing across even further...

OUT	--J18-	--J19-	--J20-	--J21-	--J22-	--J23-	--J24-	--J25-	--J26-
92	0.000001	--J19-							
93	0.000003	0.000001	--J20-						
94	0.000012	0.000003	0.000001	--J21-					
95	0.000048	0.000013	0.000003	0.000001	--J22-				
96	0.000182	0.000052	0.000014	0.000004	0.000001	--J23-			
97	0.000658	0.000209	0.000063	0.000018	0.000005	0.000001	--J24-		
98	0.002237	0.000792	0.000263	0.000082	0.000024	0.000007	0.000002	--J25-	--J26-
99	0.007075	0.002801	0.001038	0.000361	0.000119	0.000037	0.000011	0.000003	0.000001

Note - Actually at Out=0, J0=1 and J1=Zero=J2=J3 etc but the imprecision of Mod.Index previously plus some calculation roundings will cause some small errors.
The numbers near Out=Zero tend to be a bit out. I recommend using only up to 3 decimal places, otherwise you'll go mad. Even at Out=95, you'll be calculating to 15 orders of Sidebands.

DX-21 Output level ~to~ Bessel function [Jn] Table - up to 4 decimal places.

OUT	-JO--	--J1--	--J2--	--J3--	OUT	--J0--	--J1--	--J2--	--J3--	--J4--	--J5--	--J6--
0	1.0000	0.0002	0.0000		32	0.9986	0.0378	0.0007	0.0000			
1	1.0000	0.0003	0.0000		33	0.9983	0.0412	0.0009	0.0000			
2	1.0000	0.0005	0.0000		34	0.9980	0.0450	0.0010	0.0000			
3	1.0000	0.0006	0.0000		35	0.9976	0.0490	0.0012	0.0000			
4	1.0000	0.0009	0.0000		36	0.9971	0.0535	0.0014	0.0000			
5	1.0000	0.0012	0.0000		37	0.9966	0.0583	0.0017	0.0000			
6	1.0000	0.0015	0.0000		38	0.9960	0.0635	0.0020	0.0000	--J4--		
7	1.0000	0.0018	0.0000		39	0.9952	0.0693	0.0024	0.0001	0.0000		
8	1.0000	0.0022	0.0000		40	0.9943	0.0755	0.0029	0.0001	0.0000		
9	1.0000	0.0026	0.0000		41	0.9932	0.0823	0.0034	0.0001	0.0000		
10	1.0000	0.0031	0.0000		42	0.9919	0.0897	0.0040	0.0001	0.0000		
11	1.0000	0.0036	0.0000		43	0.9904	0.0977	0.0048	0.0002	0.0000		
12	1.0000	0.0043	0.0000		44	0.9886	0.1064	0.0057	0.0002	0.0000		
13	1.0000	0.0052	0.0000		45	0.9864	0.1160	0.0068	0.0003	0.0000		
14	1.0000	0.0061	0.0000		46	0.9839	0.1263	0.0081	0.0003	0.0000		
15	0.9999	0.0073	0.0000		47	0.9808	0.1375	0.0096	0.0004	0.0000		
16	0.9999	0.0080	0.0000		48	0.9772	0.1497	0.0114	0.0006	0.0000		
17	0.9999	0.0087	0.0000	--J3--	49	0.9729	0.1629	0.0135	0.0007	0.0000		
18	0.9999	0.0103	0.0001	0.0000	50	0.9678	0.1772	0.0160	0.0010	0.0000	--J5--	
19	0.9999	0.0113	0.0001	0.0000	51	0.9618	0.1926	0.0190	0.0012	0.0001	0.0000	
20	0.9998	0.0134	0.0001	0.0000	52	0.9547	0.2092	0.0226	0.0016	0.0001	0.0000	
21	0.9998	0.0146	0.0001	0.0000	53	0.9462	0.2272	0.0268	0.0021	0.0001	0.0000	
22	0.9997	0.0159	0.0001	0.0000	54	0.9362	0.2465	0.0317	0.0027	0.0002	0.0000	
23	0.9997	0.0174	0.0002	0.0000	55	0.9244	0.2671	0.0376	0.0035	0.0002	0.0000	
24	0.9996	0.0189	0.0002	0.0000	56	0.9104	0.2891	0.0445	0.0045	0.0003	0.0000	
25	0.9996	0.0206	0.0002	0.0000	57	0.8939	0.3125	0.0526	0.0058	0.0005	0.0000	
26	0.9995	0.0225	0.0003	0.0000	58	0.8745	0.3373	0.0621	0.0075	0.0007	0.0000	--J6--
27	0.9994	0.0245	0.0003	0.0000	59	0.8516	0.3632	0.0732	0.0097	0.0010	0.0001	0.0000
28	0.9993	0.0268	0.0004	0.0000	60	0.8248	0.3902	0.0862	0.0125	0.0014	0.0001	0.0000
29	0.9991	0.0292	0.0004	0.0000	61	0.7935	0.4179	0.1013	0.0161	0.0019	0.0002	0.0000
30	0.9990	0.0318	0.0005	0.0000	62	0.7570	0.4460	0.1188	0.0206	0.0027	0.0003	0.0000
31	0.9988	0.0347	0.0006	0.0000	63	0.7146	0.4740	0.1390	0.0264	0.0037	0.0004	0.0000

OUT	--J0--	--J1--	--J2--	--J3--	--J4--	--J5--	--J6--	--J7--	--J8--	--J9--	--J10-
64	0.6655	0.5011	0.1620	0.0337	0.0052	0.0006	0.0001	0.0000			
65	0.6091	0.5265	0.1881	0.0430	0.0073	0.0010	0.0001	0.0000			
66	0.5448	0.5489	0.2173	0.0546	0.0101	0.0015	0.0002	0.0000			
67	0.4720	0.5668	0.2497	0.0690	0.0140	0.0022	0.0003	0.0000	--J8--		
68	0.3905	0.5785	0.2849	0.0869	0.0193	0.0034	0.0005	0.0001	0.0000		
69	0.3004	0.5817	0.3224	0.1086	0.0266	0.0051	0.0008	0.0001	0.0000		
70	0.2026	0.5741	0.3611	0.1349	0.0363	0.0077	0.0013	0.0002	0.0000	--J9--	
71	0.0985	0.5528	0.3992	0.1661	0.0493	0.0114	0.0022	0.0004	0.0001	0.0000	
72	-0.0091	0.5153	0.4346	0.2022	0.0664	0.0169	0.0035	0.0006	0.0001	0.0000	
73	-0.1162	0.4590	0.4637	0.2431	0.0885	0.0249	0.0057	0.0011	0.0002	0.0000	--J10-
74	-0.2171	0.3822	0.4824	0.2876	0.1166	0.0362	0.0092	0.0020	0.0004	0.0001	0.0000
75	-0.3042	0.2846	0.4854	0.3335	0.1514	0.0521	0.0145	0.0034	0.0007	0.0001	0.0000
76	-0.3688	0.1684	0.4671	0.3770	0.1931	0.0740	0.0228	0.0059	0.0013	0.0003	0.0000
77	-0.4009	0.0390	0.4218	0.4126	0.2409	0.1032	0.0352	0.0101	0.0025	0.0005	0.0001
78	-0.3912	-0.0938	0.3452	0.4327	0.2921	0.1408	0.0536	0.0169	0.0046	0.0011	0.0002
79	-0.3333	-0.2152	0.2364	0.4281	0.3417	0.1872	0.0797	0.0279	0.0084	0.0022	0.0005
80	-0.2268	-0.3062	0.1004	0.3891	0.3814	0.2407	0.1154	0.0451	0.0150	0.0043	0.0011
81	-0.0815	-0.3457	-0.0494	0.3084	0.3996	0.2966	0.1618	0.0710	0.0262	0.0084	0.0024
82	0.0797	-0.3164	-0.1895	0.1848	0.3820	0.3455	0.2178	0.1080	0.0446	0.0159	0.0050
83	0.2203	-0.2124	-0.2879	0.0291	0.3157	0.3728	0.2777	0.1575	0.0733	0.0291	0.0101
84	0.2961	-0.0495	-0.3105	-0.1318	0.1951	0.3596	0.3297	0.2178	0.1153	0.0515	0.0200
85	0.2700	0.1282	-0.2357	-0.2544	0.0314	0.2880	0.3541	0.2806	0.1717	0.0870	0.0380
86	0.1354	0.2530	-0.0733	-0.2890	-0.1394	0.1521	0.3261	0.3281	0.2377	0.1386	0.0685
87	-0.0616	0.2572	0.1195	-0.2034	-0.2569	-0.0278	0.2255	0.3324	0.2982	0.2045	0.1161
88	-0.2206	0.1190	0.2452	-0.0178	-0.2562	-0.1937	0.0562	0.2634	0.3243	0.2721	0.1811
89	-0.2309	-0.0940	0.2131	0.1746	-0.1139	-0.2609	-0.1330	0.1099	0.2785	0.3119	0.2527
90	-0.0623	-0.2294	0.0225	0.2372	0.1010	-0.1671	-0.2460	-0.0891	0.1377	0.2804	0.3002
91	0.1575	-0.1545	-0.1821	0.0966	0.2282	0.0487	-0.1894	-0.2296	-0.0664	0.1451	0.2742
92	0.2029	0.0799	-0.1913	-0.1357	0.1318	0.2127	0.0233	-0.1922	-0.2197	-0.0643	0.1352
93	-0.0027	0.2063	0.0303	-0.1982	-0.1099	0.1394	0.2032	0.0238	-0.1809	-0.2175	-0.0810
94	-0.1935	0.0342	0.1977	0.0143	-0.1924	-0.1088	0.1257	0.2013	0.0473	-0.1549	-0.2184
95	-0.0559	-0.1824	0.0353	0.1904	0.0290	-0.1774	-0.1288	0.0904	0.2000	0.0896	-0.1092
96	0.1751	-0.0423	-0.1794	0.0053	0.1811	0.0694	-0.1452	-0.1594	0.0301	0.1842	0.1410
97	0.0135	0.1734	0.0030	-0.1728	-0.0520	0.1531	0.1245	-0.0825	-0.1791	-0.0531	0.1338
98	-0.1604	-0.0470	0.1563	0.0741	-0.1370	-0.1217	0.0842	0.1655	0.0163	-0.1542	-0.1368
99	0.1120	-0.1109	-0.1208	0.0917	0.1427	-0.0462	-0.1611	-0.0307	0.1440	0.1223	-0.0564

continuing across...

OUT	--J11-	--J12-	--J13-	--J14-	--J15-	--J16-	--J17-	--J18-	--J19-	--J20-	--J21-
78	0.0000										
79	0.0001	0.0000	--J13-								
80	0.0003	0.0001	0.0000								
81	0.0006	0.0001	0.0000	--J14-							
82	0.0014	0.0004	0.0001	0.0000	--J15-						
83	0.0031	0.0009	0.0002	0.0001	0.0000						
84	0.0069	0.0021	0.0006	0.0002	0.0000	--J16-					
85	0.0146	0.0050	0.0016	0.0005	0.0001	0.0000	--J17-				
86	0.0296	0.0114	0.0040	0.0013	0.0004	0.0001	0.0000	--J18-			
87	0.0568	0.0246	0.0096	0.0034	0.0011	0.0003	0.0001	0.0000	--J19-		
88	0.1017	0.0499	0.0218	0.0086	0.0031	0.0010	0.0003	0.0001	0.0000	--J20-	
89	0.1664	0.0938	0.0465	0.0207	0.0084	0.0031	0.0011	0.0003	0.0001	0.0000	--J21-
90	0.2407	0.1593	0.0911	0.0462	0.0211	0.0088	0.0034	0.0012	0.0004	0.0001	0.0000
91	0.2913	0.2358	0.1590	0.0932	0.0487	0.0231	0.0100	0.0040	0.0015	0.0005	0.0002
92	0.2617	0.2849	0.2373	0.1652	0.1004	0.0545	0.0269	0.0122	0.0051	0.0020	0.0007
93	0.1090	0.2415	0.2789	0.2437	0.1777	0.1130	0.0643	0.0332	0.0158	0.0070	0.0029
94	-0.1131	0.0657	0.2098	0.2691	0.2525	0.1957	0.1318	0.0793	0.0434	0.0218	0.0102
95	-0.2125	-0.1539	0.0047	0.1608	0.2486	0.2589	0.2176	0.1573	0.1012	0.0590	0.0316
96	-0.0387	-0.1849	-0.1903	-0.0704	0.0886	0.2076	0.2541	0.2383	0.1885	0.1313	0.0825
97	0.1798	0.0533	-0.1192	-0.2000	-0.1458	-0.0069	0.1353	0.2246	0.2473	0.2200	0.1691
98	0.0355	0.1707	0.1422	-0.0102	-0.1546	-0.1911	-0.1106	0.0278	0.1541	0.2263	0.2386
99	-0.1672	-0.0900	0.0813	0.1741	0.1126	-0.0396	-0.1631	-0.1810	-0.0962	0.0356	0.1528

continuing even further across...

OUT	--J22-	--J23-	--J24-	--J25-	--J26-	--J27-	--J28-	--J29-	--J30-	--J31-	--J32-
91	0.0001	0.0000									
92	0.0003	0.0001	0.0000	--J25-							
93	0.0011	0.0004	0.0001	0.0000	--J26-	--J27-					
94	0.0045	0.0018	0.0007	0.0003	0.0001	0.0000	--J28-	--J29-			
95	0.0157	0.0073	0.0032	0.0013	0.0005	0.0002	0.0001	0.0000	--J30-	--J31-	
96	0.0475	0.0253	0.0126	0.0059	0.0026	0.0011	0.0004	0.0002	0.0001	0.0000	--J32-
97	0.1161	0.0725	0.0418	0.0225	0.0113	0.0054	0.0024	0.0010	0.0004	0.0002	0.0001
98	0.2086	0.1596	0.1099	0.0694	0.0406	0.0222	0.0114	0.0056	0.0026	0.0011	0.0005
99	0.2198	0.2319	0.2048	0.1591	0.1118	0.0722	0.0434	0.0244	0.0130	0.0065	0.0031

continuing yet even further across...

OUT	--J33---J34-	--J35-	--J36-	
97	0.0000			
98	0.0002	0.0001	0.0000	
99	0.0014	0.0006	0.0003	0.0001

Note - Actually at Out=0, J0=1 and J1=Zero=J2=J3 etc but the imprecision of Mod.Index previously plus some calculation roundings will cause some small errors.

The numbers near Out=Zero tend to be a bit out.

Because the DX-21 type synths can attain Mod.Index=25, the number of Sidebands can go up to very high orders.

CX-5 Output level ~to~ Bessel function [Jn] Table - up to 4 decimal places.

OUT	--J0--	--J1--	--J2--
0	1.0000	0.0001	0.0000
1	1.0000	0.0001	0.0000
2	1.0000	0.0001	0.0000
3	1.0000	0.0001	0.0000
4	1.0000	0.0001	0.0000
5	1.0000	0.0002	0.0000
6	1.0000	0.0002	0.0000
7	1.0000	0.0002	0.0000
8	1.0000	0.0002	0.0000
9	1.0000	0.0002	0.0000
10	1.0000	0.0002	0.0000
11	1.0000	0.0003	0.0000
12	1.0000	0.0003	0.0000
13	1.0000	0.0003	0.0000
14	1.0000	0.0004	0.0000
15	1.0000	0.0004	0.0000
16	1.0000	0.0004	0.0000
17	1.0000	0.0005	0.0000
18	1.0000	0.0005	0.0000
19	1.0000	0.0005	0.0000
20	1.0000	0.0006	0.0000
21	1.0000	0.0006	0.0000
22	1.0000	0.0007	0.0000
23	1.0000	0.0008	0.0000
24	1.0000	0.0008	0.0000
25	1.0000	0.0009	0.0000
26	1.0000	0.0010	0.0000
27	1.0000	0.0011	0.0000
28	1.0000	0.0012	0.0000
29	1.0000	0.0013	0.0000
30	1.0000	0.0014	0.0000
31	1.0000	0.0015	0.0000
32	1.0000	0.0017	0.0000
33	1.0000	0.0018	0.0000
34	1.0000	0.0020	0.0000
35	1.0000	0.0022	0.0000
36	1.0000	0.0024	0.0000
37	1.0000	0.0026	0.0000
38	1.0000	0.0028	0.0000
39	1.0000	0.0031	0.0000
40	1.0000	0.0033	0.0000
41	1.0000	0.0036	0.0000
42	1.0000	0.0040	0.0000
43	1.0000	0.0043	0.0000
44	1.0000	0.0047	0.0000
45	1.0000	0.0052	0.0000
46	1.0000	0.0056	0.0000

OUT	--J0--	--J2--	--J3--	--J4--	--J5--	--J6--
47	1.0000	0.0061	0.0000			
48	1.0000	0.0067	0.0000			
49	0.9999	0.0073	0.0000			
50	0.9999	0.0080	0.0000			
51	0.9999	0.0087	0.0000			
52	0.9999	0.0095	0.0000	--J4--		
53	0.9999	0.0103	0.0001	0.0000		
54	0.9999	0.0113	0.0001	0.0000		
55	0.9998	0.0123	0.0001	0.0000		
56	0.9998	0.0134	0.0001	0.0000		
57	0.9998	0.0146	0.0001	0.0000		
58	0.9997	0.0159	0.0001	0.0000		
59	0.9997	0.0174	0.0002	0.0000		
60	0.9996	0.0189	0.0002	0.0000		
61	0.9996	0.0206	0.0002	0.0000		
62	0.9995	0.0225	0.0003	0.0000		
63	0.9994	0.0245	0.0003	0.0000		
64	0.9993	0.0268	0.0004	0.0000		
65	0.9991	0.0292	0.0004	0.0000		
66	0.9990	0.0318	0.0005	0.0000		
67	0.9988	0.0347	0.0006	0.0000		
68	0.9986	0.0378	0.0007	0.0000		
69	0.9983	0.0412	0.0009	0.0000		
70	0.9980	0.0450	0.0010	0.0000		
71	0.9976	0.0490	0.0012	0.0000		
72	0.9971	0.0535	0.0014	0.0000		
73	0.9966	0.0583	0.0017	0.0000		
74	0.9960	0.0635	0.0020	0.0000	--J5--	
75	0.9952	0.0693	0.0024	0.0001	0.0000	
76	0.9943	0.0755	0.0029	0.0001	0.0000	
77	0.9932	0.0823	0.0034	0.0001	0.0000	
78	0.9919	0.0897	0.0040	0.0001	0.0000	
79	0.9904	0.0977	0.0048	0.0002	0.0000	
80	0.9886	0.1064	0.0057	0.0002	0.0000	
81	0.9864	0.1160	0.0068	0.0003	0.0000	
82	0.9839	0.1263	0.0081	0.0003	0.0000	
83	0.9808	0.1375	0.0096	0.0004	0.0000	
84	0.9772	0.1497	0.0114	0.0006	0.0000	
85	0.9729	0.1629	0.0135	0.0007	0.0000	
86	0.9678	0.1772	0.0160	0.0010	0.0000	--J6--
87	0.9618	0.1926	0.0190	0.0012	0.0001	0.0000
88	0.9547	0.2092	0.0226	0.0016	0.0001	0.0000
89	0.9462	0.2272	0.0268	0.0021	0.0001	0.0000
90	0.9362	0.2465	0.0317	0.0027	0.0002	0.0000
91	0.9244	0.2671	0.0376	0.0035	0.0002	0.0000
92	0.9104	0.2891	0.0445	0.0045	0.0003	0.0000
93	0.8939	0.3125	0.0526	0.0058	0.0005	0.0000
94	0.8745	0.3373	0.0621	0.0075	0.0007	0.0000

OUT	--J0--	--J1--	--J2--	--J3--	--J4--	--J5--	--J6--	--J7--	--J8--	--J9--	--J10-
95	0.8516	0.3632	0.0732	0.0097	0.0010	0.0001	0.0000				
96	0.8248	0.3902	0.0862	0.0125	0.0014	0.0001	0.0000				
97	0.7935	0.4179	0.1013	0.0161	0.0019	0.0002	0.0000				
98	0.7570	0.4460	0.1188	0.0206	0.0027	0.0003	0.0000				
99	0.7146	0.4740	0.1390	0.0264	0.0037	0.0004	0.0000	--J7--			
100	0.6655	0.5011	0.1620	0.0337	0.0052	0.0006	0.0001	0.0000			
101	0.6091	0.5265	0.1881	0.0430	0.0073	0.0010	0.0001	0.0000			
102	0.5448	0.5489	0.2173	0.0546	0.0101	0.0015	0.0002	0.0000			
103	0.4720	0.5668	0.2497	0.0690	0.0140	0.0022	0.0003	0.0000	--J8--		
104	0.3905	0.5785	0.2849	0.0869	0.0193	0.0034	0.0005	0.0001	0.0000		
105	0.3004	0.5817	0.3224	0.1086	0.0266	0.0051	0.0008	0.0001	0.0000		
106	0.2026	0.5741	0.3611	0.1349	0.0363	0.0077	0.0013	0.0002	0.0000	--J9--	
107	0.0985	0.5528	0.3992	0.1661	0.0493	0.0114	0.0022	0.0004	0.0001	0.0000	
108	-0.0091	0.5153	0.4346	0.2022	0.0664	0.0169	0.0035	0.0006	0.0001	0.0000	
109	-0.1162	0.4590	0.4637	0.2431	0.0885	0.0249	0.0057	0.0011	0.0002	0.0000	--J10-
110	-0.2171	0.3822	0.4824	0.2876	0.1166	0.0362	0.0092	0.0020	0.0004	0.0001	0.0000
111	-0.3042	0.2846	0.4854	0.3335	0.1514	0.0521	0.0145	0.0034	0.0007	0.0001	0.0000
112	-0.3688	0.1684	0.4671	0.3770	0.1931	0.0740	0.0228	0.0059	0.0013	0.0003	0.0000
113	-0.4009	0.0390	0.4218	0.4126	0.2409	0.1032	0.0352	0.0101	0.0025	0.0005	0.0001
114	-0.3912	-0.0938	0.3452	0.4327	0.2921	0.1408	0.0536	0.0169	0.0046	0.0011	0.0002
115	-0.3333	-0.2152	0.2364	0.4281	0.3417	0.1872	0.0797	0.0279	0.0084	0.0022	0.0005
116	-0.2268	-0.3062	0.1004	0.3891	0.3814	0.2407	0.1154	0.0451	0.0150	0.0043	0.0011
117	-0.0815	-0.3457	-0.0494	0.3084	0.3996	0.2966	0.1618	0.0710	0.0262	0.0084	0.0024
118	0.0797	-0.3164	-0.1895	0.1848	0.3820	0.3455	0.2178	0.1080	0.0446	0.0159	0.0050
119	0.2203	-0.2124	-0.2879	0.0291	0.3157	0.3728	0.2777	0.1575	0.0733	0.0291	0.0101
120	0.2961	-0.0495	-0.3105	-0.1318	0.1951	0.3596	0.3297	0.2178	0.1153	0.0515	0.0200
121	0.2700	0.1282	-0.2357	-0.2544	0.0314	0.2880	0.3541	0.2806	0.1717	0.0870	0.0380
122	0.1354	0.2530	-0.0733	-0.2890	-0.1394	0.1521	0.3261	0.3281	0.2377	0.1386	0.0685
123	-0.0616	0.2572	0.1195	-0.2034	-0.2569	-0.0278	0.2255	0.3324	0.2982	0.2045	0.1161
124	-0.2206	0.1190	0.2452	-0.0178	-0.2562	-0.1937	0.0562	0.2634	0.3243	0.2721	0.1811
125	-0.2309	-0.0940	0.2131	0.1746	-0.1139	-0.2609	-0.1330	0.1099	0.2785	0.3119	0.2527
126	-0.0623	-0.2294	0.0225	0.2372	0.1010	-0.1671	-0.2460	-0.0891	0.1377	0.2804	0.3002
127	0.1575	-0.1545	-0.1821	0.0966	0.2282	0.0487	-0.1894	-0.2296	-0.0664	0.1451	0.2742

continuing across...

OUT	--J11-	--J12-	--J13-	--J14-	--J15-	--J16-	--J17-	--J18-	--J19-	--J20-	--J21-	--J22-
114	0.0000											
115	0.0001	0.0000	--J13-									
116	0.0003	0.0001	0.0000									
117	0.0006	0.0001	0.0000	--J14-								
118	0.0014	0.0004	0.0001	0.0000	--J15-							
119	0.0031	0.0009	0.0002	0.0001	0.0000							
120	0.0069	0.0021	0.0006	0.0002	0.0000	--J16-						
121	0.0146	0.0050	0.0016	0.0005	0.0001	0.0000	--J17-					
122	0.0296	0.0114	0.0040	0.0013	0.0004	0.0001	0.0000	--J18-				
123	0.0568	0.0246	0.0096	0.0034	0.0011	0.0003	0.0001	0.0000	--J19-			
124	0.1017	0.0499	0.0218	0.0086	0.0031	0.0010	0.0003	0.0001	0.0000	--J20-		
125	0.1664	0.0938	0.0465	0.0207	0.0084	0.0031	0.0011	0.0003	0.0001	0.0000	--J21-	
126	0.2407	0.1593	0.0911	0.0462	0.0211	0.0088	0.0034	0.0012	0.0004	0.0001	0.0000	--J22-
127	0.2913	0.2358	0.1590	0.0932	0.0487	0.0231	0.0100	0.0040	0.0015	0.0005	0.0002	0.0001

Note - Actually at Out=0, J0=1 and J1=Zero=J2=J3 etc but the imprecision of Mod.Index previously plus some calculation roundings will cause some small errors.

The numbers near Out=Zero tend to be a bit out.

The range of Bessel values for the CX-5 type synths are similar to the DX-7 type but the Mod.Index curve is slightly different.

Recommended Reading

Here are some links and material I've found for FM Synthesis and related information.

http://www.sfu.ca/sca/Manuals/fm/FM_Tutorial.html
http://www.sfu.ca/~truax/fmtut.html
http://www.esm.rochester.edu/www/onlinedocs/allan.cs/chapter3.html
http://cis.poly.edu/cs240/notes6.htm
http://www.telecommunication.msu.edu/classes/tc201/slides/fm/index.htm
http://www.neuroinformatik.ruhr-uni-bochum.de/ini/PEOPLE/heja/sy-prog/node55.html
http://ccrma-www.stanford.edu/CCRMA/Software/clm/clm-manual/fm.html

I haven't personally read this stuff, but they come highly recommended.

The Synthesis of Complex Audio Spectra by Means of Frequency Modulation [John M Chowning & Max Mathews] - Audio Engineering Society Journal
- Vol.21/ No.7 (1973): pg 526-534.

FM Theory and Applications by Musicians for Musicians [John M Chowning & Dave Bristow] - Yamaha Music Foundation, Tokyo (1986) ISBN 4-636-17482-8

The Simulation of Natural Instrument Tones using Frequency Modulation with a Complex Modulating Wave [Bill Schottstaedt] - Computer Music Journal - Vol.1, No.4 (1977) : pg 46-50.

A Derivation of the Spectrum of FM with a Complex Modulating Wave [Marc LeBrun] - Computer Music Journal - Vol.1/ No.4 (1977): pg 51-52.

The F.M. legend - a personal history

For me, it all started one day in mid'1984 when I walked into my friendly neighbourhood music-inst shop (actually, it was Soho Soundhouse, London) and there was a buzz in the air. The sales-guy says "You're here to try out the DX-7, right?". I, of course, didn't know what he was talking about and enquired about the price. It was out of my reach (obviously). Not to be deterred, the sales-guy instead plonks me in front of a Yamaha DX-9 and hands me some headphones. I started playing and... aaaahh, heaven!

It's hard to describe what I heard (bearing in mind it's my first time hearing F.M. synthesis). You have to understand that, up to this point, synths were all about strings and brass. Occasionally, you'd have a few plinky plonky xylo-sounds (heck, the MKS-10 was considered realistic) but percussives, vibes, pianos etc were elusive (ie non-existent). But right there in front of me, in the form of a DX-9, was the holy grail. And, to make matters worse, there weren't any knobs or sliders or anything in fact which gave any inkling as to how this synth worked.

I was hooked! I took the plunge and bought a DX-7 in Nov'84 and later a CX-5 in Jan'85. Unfortunately, programming these FM synths was an absolute nightmare. Nothing was fast and nothing was easy. It really wasn't intuitive at all and the manual wasn't exactly that helpful. But I was determined to master this beast. Learning to program the DX-7 was a slow and tedious process.

But one day in late 1984, humanity was saved by a fellow synth-enthusiast called Tony Wride. Fed-up and tired with struggling alone with his DX-7, he mooted the idea of a "DX-club" in a letter to the magazine "Electronics And Music Maker". This caused a huge stirring of support from the public (DX synth owners), the media (music mags) and Yamaha too. Thus was born the DX-Owners Club.

It was the DX-Owners Club which took FM programming to new heights. Via its newsletter, we began sharing patches/ sounds (one patch called "Wurlitzer" was really popular) and programming techniques (excellent articles by Ken Campbell). Part of Tony Wride's vision was also to have a network of co-ordinators who anyone could telephone for help and advice (you'll find listed under Area Co-ordinator for London W2 is "Yahaya 01-221-5314" which is me). Beyond the popular newsletter (these typed-up/ hand-drawn photocopy newsletters were inspirational), the club also organised get-together seminars bringing programmers together to meet experts like Dave Bristow to discuss FM in depth (I remember that fixed-frequency operators was a big topic). FM ruled and the DX-7 was king... Life was good!

But as life goes on, reality takes its hold... Tony's job in the RAF gave him less time to run the club. Eventually, the club was handed over to(surprise surprise) Yamaha who appointed Martin Tennant (not to be confused with Martin Russ) to run the re-named X-Series Owners Club. With Yamaha's backing (ie money), the newsletter became a regular magazine (with pictures and all) and everything was taken to a more polished level.

The X-Series Owners Club was good... but with the change of management of the club, came a change in objectivity. You see, us members may all be dx-synth enthusiasts but we didn't work for Yamaha (the old newsletter would include info on non-Yamaha products as well). I remember an interesting session where Yamaha was launching their DX-5 while Tony was happily proposing to just add a TX-7 to a DX-7 (ie half the cost). Ah, well! Nevermind.

As far as I know, the magazine continued until around mid'1987.

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